

# Pseudo-random binary injection of levitons for electron quantum optics

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The recent realization of single electron sources lets envision performing electron quantum optics experiments, where electrons can be viewed as flying qubits propagating in a ballistic conductor. To date, all electron sources operate in a periodic electron injection mode leading to energy spectrum singularities in various physical observable which sometime hides the bare nature of physical effects. To go beyond, we propose a spread-spectrum approach where electron flying qubits are injected in a non-periodic manner following a pseudo-random binary bit pattern. Extending the Floquet scattering theory approach from periodic to spread-spectrum drive, the shot noise of pseudo-random binary sequences of single electron injection can be calculated for leviton and non-leviton sources. In particular, the spread spectrum approach is shown to provide a better knowledge of electronic Hong Ou Mandel correlations and clarifies the role of the dynamical orthogonality catastrophe for non-integer charge injection.

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We consider the pseudo-random binary injection of electrons in a quantum conductor using series of voltage pulses enabling their use in further work as flying qubits [1, 2] in Electron Quantum Optics.

The goal of Electron quantum optics is to perform quantum operations similar to those done with photons in quantum optics but with electrons. For ballistic quantum conductors, electron beam-splitters are available using Quantum Point Contacts (QPC) to partition single electron propagating on single quantum channels. Combining two QPC beam-splitters provide electron analogs of optical Mach-Zehnder [3, 4] and Fabry-Pérot interferometers. A full Electron Quantum Optics requires the electronic analog of single photon sources. Several approaches have been used for the realization of the on-demand injection of single electrons [5, 6, 8–12]. Here, we consider the voltage pulse source [12, 13] which is simpler to built and operate. It is based on voltage pulses applied on a contact injecting a charge in the ballistic conductor. Moreover, for Lorentzian voltage pulses, electrons are injected in the form of a remarkable minimal excitation state [14–18] called a leviton [12]. Synchronizing the injection of levitons from different contacts and letting them interfere in a quantum conductor lets envisage flying qubit operation in a simple way. This approach has already lead to new quantum experiments where single electron partitioning [12, 19], electronic Hong Ou Mandel interference [12, 20], or single electron quantum tomography [13] have been shown.

For practical ease of operation and calculation, only periodic injection of electrons have been considered to date. However, for driving frequency  $\nu = 1/T$ , step-wise discontinuities appear in the energy density or in the electronic Wigner function [13, 22] of periodic levitons at energies energy multiple of  $h\nu$  [21]. This is also reflected by singularities in the shot noise of levitons par-

tioned by a QPC. These singularities may prevent to understand if the phenomena observed result from periodicity or from the nature of the injected charge. Injecting unique single electron pulses could be the alternative but the present lack of reliable single electron detectors prevents this approach to be useful. A closer approach is the non-periodic injection of electrons following a pseudo-random binary bit pattern  $\{b_k\}$  where at each time  $t = kT$  an (or no) electron injected if  $b_k = 1$  (or 0). A direct result is to spread the energy spectrum and we will show that the electron energy spectrum is made from a continuous part directly related to the variance of bit  $b_k$  while weaker spectral discontinuities remains directly related to regular bit sequence. Comparing periodic and random injection we also enable us to better show the effect of the dynamical orthogonality catastrophe predicted in [14, 15] for non integer charge injection and to quantify the amount of electron-hole pairs created while injecting electrons with non-Lorentzian pulses. Another advantage of non-periodic injection is found in the case of electronic HOM interferometry which gives the time correlation function of the electron wavefunction  $|\langle\psi(\tau)|\psi(0)\rangle|^2$ . As we will later show, this allows for a better exploration of the tails of the single electron (leviton) wavefunction while periodicity limits this information to a half period time-scale:  $\tau \leq T/2$ . In a broader perspective, non-periodic injection belongs to the class of spread-spectrum techniques. The response of quantum systems to spread spectrum excitations may be viewed as a new field which, except in quantum communication, has not yet been explored and which could shed a new (white) light on quantum effects.

Before presenting our approach to deal with this new situation, we recall the main results of the Floquet scattering approach [23] for periodic excitation and then extend the Floquet approach to non-periodic drive. We

consider a periodic voltage  $V_L(t)$  applied to, say the left, contact of a ballistic conductor while the opposite (right) contact is kept to ground ( $V_R = 0$ ). According to the scattering approach developed by Moskalets and Büttiker [23], an electron emitted at energy  $\varepsilon$  below the Fermi energy  $\mu_L$  from the left contact and experiencing the ac excitation has its phase  $\phi(t)$  modulated by the voltage, with  $\phi(t) = \int_{-\infty}^t eV(t')dt'/\hbar$ . The time dependence breaking energy conservation, the electron exiting the left contact is scattered into a superposition of quantum states at energies  $\varepsilon + l\hbar\nu$ , where  $l$  is an integer. The scattering amplitudes connecting initial and final energies are the photo-absorption (emission) amplitudes  $p_l$  which forms the elements of the so-called unitary Floquet scattering matrix, where positive (negative)  $l$  means absorption (emission). The  $p_l$  are the Fourier transforms of the phase term, namely  $p_l = \frac{1}{T} \int_0^T \exp(-i\phi(t)) \exp(i2\pi l\nu t) dt$ . From unitarity the  $p_l$  obey the following useful identity  $\sum_l p_{l+k}^* p_l = \delta_{k,0}$ , where  $*$  denote the complex conjugate. To calculate transport properties, the standard annihilation fermionic operators  $\hat{a}_L(\varepsilon)$  which operate on the equilibrium states of the left contact are replaced by new annihilation fermion operators  $\hat{\tilde{a}}_L(\varepsilon)$ , with  $\hat{\tilde{a}}_L(\varepsilon) = \sum_l p_l \hat{a}_L(\varepsilon - l\hbar\nu)$ . This substitution on transport formula provides direct expression for the mean photo-assisted current  $\tilde{I}$  and mean photo-assisted shot noise  $\tilde{S}_I$  of a conductor related to their dc transport expressions  $I$  and  $S_I$ . Adding an extra dc voltage  $V_{dc}$  to the ac voltage  $V(t)$  for generality, gives the following expressions:

$$\tilde{I} = \sum_l P_l I(V_{dc} + l\hbar\nu) \quad (1)$$

$$\tilde{S}_I = \sum_l P_l S_I(V_{dc} + l\hbar\nu) \quad (2)$$

where  $P_l = |p_l|^2$  is the probability to absorb or emit  $l$  photons. A typical application is a single channel conductor with a QPC in its middle transmitting electrons with transmission probability  $D$ . For energy independent transmission, linear I-V characteristics, remarkably  $\tilde{I}(V_{dc}) = I(V_{dc}) = (e^2/h)V_{dc}$  as from unitarity  $\sum_l P_l = 1$ , while  $\sum_l lP_l = 0$ . By contrast, the shot noise shows a non-linear (rectification like) variation with current (or voltage):  $S_I = 2e|I|(1 - D)$ . From Eq.(2), this gives replica of the zero bias singularity in the photo-assisted shot noise  $\tilde{S}_I$  each time  $V_{dc} = l\hbar\nu$  [24–28]. The singularities result from stepwise variations of the energy distribution  $\tilde{f}(\varepsilon)$  of the periodically driven Fermi sea given by:

$$\tilde{f}(\varepsilon) = \sum_l P_l f(\varepsilon - l\hbar\nu) \quad (3)$$

where  $f(\varepsilon) = 1/(1 + \exp(\beta(\varepsilon - \mu)))$  is the Fermi Dirac distribution of electrons in the left contact at chemical

potential  $\mu = eV_{dc}$  and electronic temperature  $k_B T_e = 1/\beta$ . Terms with positive (negative)  $l$  describe electron (hole) like excitations.

As shown in [14, 15, 17] and detailed in [21], a particular situation arises when the voltage  $V(t)$  is a sum of periodic Lorentzian pulses, each introducing a single electron in the conductor from the left contact, where:

$$V(t) = \frac{\hbar}{\pi e T} \sum_{k=-\infty}^{+\infty} \frac{1}{1 + (t - kT)^2/W^2} \quad (4)$$

Here the  $P_l$  remarkably vanish for  $l < 0$  and  $\tilde{f}(\varepsilon)$  describes a pure electron excitation population with no holes. This defines a *minimal excitation states* [17] generated by the periodic train of levitons [12]. In general, an arbitrary shape (non Lorentzian) of voltage pulses gives non-zero  $P_l$  for negative  $l$  and thus generate a mixture of electron and hole excitations. This is for example the case for sine-wave voltage pulses  $V(t) = (\hbar\nu/e)(1 - \cos(2\pi\nu t))$  injecting single electrons.

We now address non periodic excitations leading to spread spectrum property. Levitons are injected following a pseudo-random sequence of binary bits  $b_k = 0, 1$  with

$$V(t) = \frac{\hbar}{\pi e T} \sum_{k=-\infty}^{+\infty} \frac{b_k}{1 + (t - kT)^2/W^2} \quad (5)$$

We expect that the photo-absorption probabilities become a continuous function of the energy. To make contact with periodic drive, we define the density of probability per unit energy as  $P(\varepsilon) = \sum_l P_l \delta(\varepsilon - l\hbar\nu)$  in the periodic regime. For the spread spectrum case, the definition of the photo absorption or emission amplitude probabilities becomes  $p(\delta\varepsilon) = \int_{-\infty}^{+\infty} dt \exp(-i\phi(t)) \exp(i\delta\varepsilon t/\hbar)$ . This describes the amplitude to find an electron, initially emitted by the reservoir at energy  $E$ , scattered to the energy  $E + \varepsilon$ . The problem of calculating  $P(\varepsilon)$  is similar to the calculation of the power spectrum density in digital communication for binary phase pulses modulation, see [29], where frequency replaces energy here. To obtain  $P(\varepsilon)$ , one needs the two-time autocorrelation function:

$$\langle C(t, t') \rangle = \langle \exp(-i\phi(t)) \exp(i\phi(t')) \rangle \quad (6)$$

where  $\langle \cdot \rangle$  means ensemble averaging over statistically independent bit patterns. One can show that  $\langle C(t, t') \rangle$  is cyclo-stationary, i.e.  $\langle C(\bar{t} + \tau/2, \bar{t} - \tau/2) \rangle$  is invariant when  $\bar{t} \rightarrow \bar{t} + T$ , where  $\bar{t} = (t + t')/2$  is the mean time and  $\tau = t - t'$  the time difference. After averaging over  $\bar{t}$  we get the time average correlation function  $\langle \overline{C(\tau)} \rangle$  whose Fourier Transform (F.T.) gives  $P(\varepsilon) = (\hbar/T) \int_{-\infty}^{\infty} \langle \overline{C(\tau)} \rangle e^{i\varepsilon\tau/\hbar} d\tau$ . Then, the current, shot noise and energy distribution expressions can be calculated by extending Eqs.1, 2 3 to the continuous case.

$$\tilde{I}(V_{dc}) = \int_{-\infty}^{+\infty} P(\varepsilon) I(V_{dc} + \varepsilon) d\varepsilon \quad (7)$$

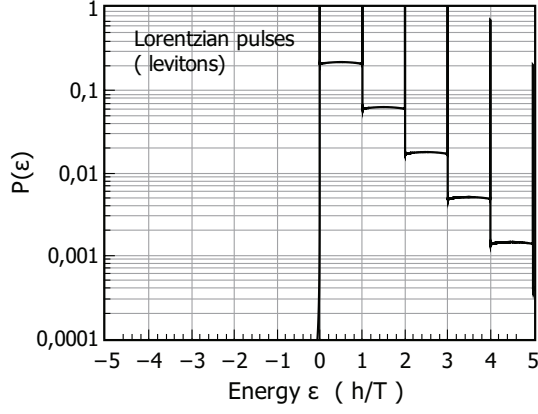


FIG. 1: The figure shows the probability  $P(\varepsilon)$ , computed from the F.T. of (11, to shift by  $\varepsilon$  the energy of electrons experiencing a pseudo random binary sequence of Lorentzian voltage pulses. The single-side band energy spectrum ( $P(\varepsilon) = 0$  for  $\varepsilon < 0$ ) is characteristic of Lorentzian voltage pulses and warranties that only electron like excitations are generated by the pulses as expected for leviton generation. This generalize to random emission the results found for periodic injection.

$$\tilde{S}_I(V_{dc}) = \int_{-\infty}^{+\infty} P(\varepsilon) S_I(V_{dc} + \varepsilon) d\varepsilon \quad (8)$$

$$\tilde{f}(\varepsilon) = \int_{-\infty}^{+\infty} P(\varepsilon') f(\varepsilon - \varepsilon') d\varepsilon' \quad (9)$$

We now give the expressions from which the physical quantities will be calculated. Remarkably, Lorentzian pulses (levitons) give rise to analytical expressions of the correlation functions. The two-time correlation function is given by:

$$\langle C(\bar{t} + \tau/2, \bar{t} - \tau/2) \rangle = \frac{\cos(2\pi\sqrt{\tau^2/4 - w^2}) - \cos(2\pi\bar{t})}{\cos 2\pi(\tau/2 - iw) - \cos(2\pi\bar{t})} \quad (10)$$

Here  $\tau$  and  $\bar{t}$  are in period units and  $w = W/T$ . As expected, it shows cyclo-stationary property. Its time average is:

$$\langle \overline{C(\tau)} \rangle = \frac{\cos(2\pi\sqrt{\tau^2/4 - w^2}) - \cos(\pi(\tau - 2iw))}{\sin(\pi(\tau - 2iw))} \quad (11)$$

The first part with the square root argument is responsible for a continuous spectrum as expected for random excitation, while the second part represent harmonic contributions giving lines in the energy spectrum reflecting the regularity of injection. Finally, the imaginary part in the argument of the sine term in the denominator indicates that  $C(\tau)$  has only poles in the upper half complex plane and no poles in the lower part and ensures that  $P(\varepsilon)$ , its F.T. vanishes for negative energies, as expected for levitons. Figure 1 shows the variations of  $P(\varepsilon)$  for  $w = W/T = 0.01$ . The energy spectrum

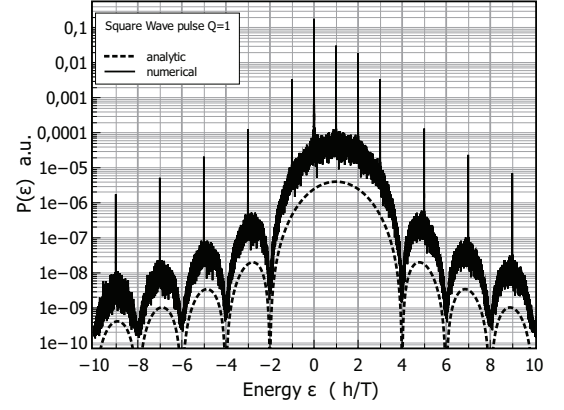


FIG. 2:  $P(\varepsilon)$  for single electrons injected with pseudo-random binary square-wave pulses. The dashed curve is the continuous part of the analytic expression (12), while the upper noisy curve represents numerical simulations for the average of 8 spectra made of uncorrelated 1024 bit patterns. For clarity a vertical shift between curves has been made by applying a different arbitrary scale factor to the curves. The double-side band energy spectrum shows the square-wave pulses, unlike levitonic pulses, generate hole like excitations as observed for periodic injection.

is continuous with spectral lines at energies multiple of  $h/T$  and extends to positive energies. For comparison we consider similar pseudo-random binary injection of single electrons but with square wave voltage pulses with  $V(t) = b_k V_0(t - kT)$  where  $V_0(t) = (2h/eT)$  if  $t \in [0, T/2]$  and  $V_0 = 0$  for  $t \in [T/2, T]$ . Expressing  $\varepsilon$  in reduced unit of  $h/T$ , we found  $P(\varepsilon)$  given by:

$$P(\varepsilon) = \frac{1}{4} \left[ \left( \frac{\sin(\pi\varepsilon/2)}{(\pi\varepsilon/2)(2-\varepsilon)} \right)^2 + \frac{9}{4}\delta(\varepsilon) + \dots \right. \\ \left. \dots + \frac{1}{4}\delta(2-\varepsilon) + \sum_{p=-\infty}^{+\infty} \frac{4}{\pi^2} \frac{1}{(2p^2-1)^2} \delta(2p+1-\varepsilon) \right] \quad (12)$$

As for the levitons, the first term gives a continuous energy spectrum followed by a series of lines at multiple of the characteristic energy  $h/T$ . One can show that the first term is  $\propto \langle b_k^2 \rangle - \langle b_k \rangle^2$  and results from the fluctuating part of the injection making the spectrum continuous, while the next terms are  $\propto \langle b_k \rangle^2$  result from the regular part of the injection. Figure 2 shows the continuous part of  $P(\varepsilon)$  calculated from Eq.(12) and the results of numerical simulations.

It is interesting to quantify the number of excitations per pulse generated by the random pulses and compare with the periodic case. The number of electrons (holes)  $N_e$  ( $N_h$ ) created per pulses is given by  $N_e = \int_0^\infty \varepsilon P(\varepsilon) d\varepsilon$  ( $N_h = \int_{-\infty}^0 (-\varepsilon) P(\varepsilon) d\varepsilon$ ) while the average charge per pulses is  $\langle q \rangle = e(N_e - N_h)$ . With these definitions, the number of extra excitations accompanying the injected charge is  $\Delta N_{exc} = N_e + N_h - \langle q \rangle$  per pulses. In terms of

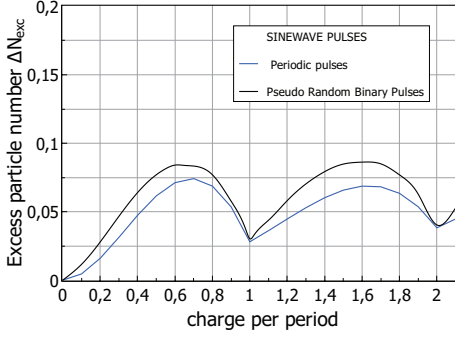


FIG. 3: The figure compares the excess number of excitations produced per period for pulses carrying a charge  $q$  for periodic and binary pulse injection. The minimum of  $N_{exc}$  is expected for integer charges while for non-integer charge it is larger as a result of the so-called Dynamical Orthogonality Catastrophe (DOC). Random binary injection giving more excitations than periodic injection is consistent with the DOC scenario as there is more time in average between pulses.

$P(\varepsilon)$ :

$$\langle q \rangle = \int_{-\infty}^{+\infty} P(\varepsilon) \varepsilon d\varepsilon \quad (13)$$

$$N_e + N_h = \int_{-\infty}^{+\infty} P(\varepsilon) |\varepsilon| d\varepsilon \quad (14)$$

$N_e + N_h$  can be measured using Shot Noise measurements, see Eq.(8), as experimentally demonstrated in [12]. Indeed, if the charge injected by the pulses in a ballistic conductor are sent to a beam-splitter of transmission  $D$ , their partitioning gives the zero temperature noise  $S_I = 2(e^2/eT)D(1-D)(N_e + N_h)$ .

We now compare various single charge voltage pulses. For all pulse shapes, the computation of (13) from  $P(\varepsilon)$  gives the same average charge  $\langle q \rangle = e/2$  per injection period, trivially resulting from equal bit probability. The Lorentzian pulses give no extra excitation  $\Delta N_{exc} = 0$  as expected while the square wave pulses give  $\Delta N_{exc}/\langle q \rangle = 0.1289$ . This is to be compared with the periodic square wave case for which it was found in [12, 21]  $\Delta N_{exc}/\langle q \rangle = 0.1082$ . We have also numerically computed the case of sine wave voltage pulse injection with  $V(t) = 2(h/eT)(1 - \cos 2\pi(t - kT)/T)$  for  $b_k = 1$  or  $V(t) = 0$  for  $b_k = 0$ . One finds  $\Delta N_{exc}/\langle q \rangle = 0.05476$  (and  $\langle q \rangle = 0.5e$ ) while for the periodic case one has  $\Delta N_{exc}/\langle q \rangle = 0.028$  (and  $\langle q \rangle = 1$ ). This confirms that sine wave pulses produce less extra excitations than square wave pulses but in both cases the pseudo-random binary injection generates more excitations per charge injected than the periodic injection. From this study we learn that the number of extra excitations is not a property of a given (single) pulse shape but depends on

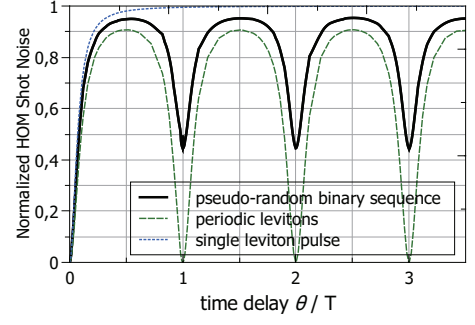


FIG. 4: Hong Ou Mandel shot noise for levitons colliding in a beam-splitter with time delay  $\theta$  and  $w = 0.035T$ . The binary (solid line), periodic (dashed line) and single pulse (dotted line) injection are compared. While periodic injection limits the information in the range  $[0, T/2]$ , information for a larger time-scale is given by the binary injection whose variation at small  $\theta$  are closer to that of a single pulse. The periodic HOM dips appear for  $\theta$  multiples of  $T$ . They results from bits (0,0) and (1,1) occurrence, each with 25% probability giving a  $\simeq 50\%$  HOM dip.

the way the pulses are injected. The discrepancy between periodic and binary injection becomes more pronounced if we consider non-integer pulses. Results are given for sine wave pulses in Figure 3 which displays the evolution of  $\Delta N_{exc}$  versus the charge per pulse for periodic and non-periodic binary injection. For non-integer charge, the number of excitations is expected to rise (logarithmically) with the mean time between charge pulses as a result of the Dynamical Orthogonality Catastrophe (DOC) defined in Refs. [14, 15]. In general, more space between pulses gives more freedom to the excited Fermi sea to create extra excitations. As a perspective, a similar study could shed light on the recently considered half-levitons [30] which are singular zero-energy fractional excitations. Finally we address electronic Hong Ou Mandel (HOM) correlations where identical binary sequences of Lorentzian pulses are applied on opposite contacts of a QPC forming a beam-splitter of transmission  $D$ . We introduce a time delay  $\theta$  between the two voltages  $V_L(t) = V^{bin}(t - \theta/2)$  and  $V_R(t) = V^{bin}(t + \theta/2)$ . The measure of the HOM interference is given by the noise  $S_I^{HOM}(\theta) \propto (1 - |\langle \psi(\theta) | \psi(0) \rangle|^2)$  observed in the current fluctuation of the output leads resulting from two-electron partitioning, as shown for periodic single electron injection in [12, 20]. The expression of the time correlation function  $\overline{C}(\tau, \theta)$  enabling calculation of  $P(\varepsilon)$  is:

$$\overline{C}(\tau, \theta) = 1 + I(\tau, \theta) + I(\tau, -\theta)^* \quad (15)$$

$$I(\tau, \theta) = \frac{i}{2} (\cos 2\pi(\tau + \theta/2 - iw) - \cos 2\pi\tau) \times \dots \frac{(\cos 2\pi(\tau + \theta/2 + iw) - \cos 2\pi\tau)}{\sin 2\pi(\tau - \theta/2 + iw) \sin 2\pi\tau \sin 2\pi(\theta/2 - iw)} \quad (16)$$

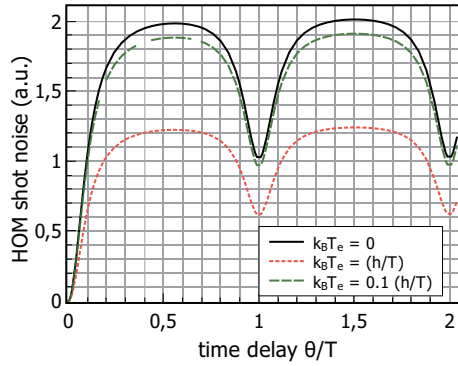


FIG. 5: Hong Ou Mandel shot noise for levitons colliding in a beam-splitter with time delay  $\theta$  and  $w = 0.05T$  at different temperatures  $T_e$ . The comparison between curves show that they are homothetic: remarkably the shape of the HOM noise curve versus time delay is not affected by temperature, except an overall reduction factor

where:

$$t^{\pm} = \left( \tau^2 + (\theta/2)^2 - w^2 \pm \sqrt{4\tau^2((\theta/2)^2 - w^2) - (w\theta/2)^2} \right)^{1/2} \quad (17)$$

The HOM noise is shown in Figure 4. For  $\theta = 0$  electrons emitted by identical sequences are undistinguishable at all time. One expects the Fermi statistics to lead to perfect antibunching and 100% noise suppression is found. We see replica of noise suppression for  $\theta = kT$ , 50% deep, which correspond to (0,0) and (1,1) bit events each occurring with 25% probability. The HOM noise of periodic levitons is shown for comparison as well as the single pulse HOM noise. The pseudo-random binary injection is in between and provides information on the leviton not limited to  $\theta \leq T/2$ . To complete this study we generalize to random injection the remarkable result found in [21] and observed in [31] for periodic injection that the HOM leviton noise shape versus  $\theta$  is not affected by temperature. This confirm the prediction of [32] that finite temperature, unlike de-coherence, does not affect HOM correlations. This is shown in Figure 5.

To conclude we have shown that random injection provides new information compared with periodic injection. Further studies may include varying the bit injection probability to provide a systematic study of the DOC or use the binary injection to simulate flying qubit operation. We hope that this work will inspire new studies exploiting spread spectrum approaches to get new information on electronic quantum systems.

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